A TECHNICAL NOTE AND RESEARCH PROSPECTUS FOR: LAMINATED GLASS MEMBRANE MODEL IMPROVEMENTS

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Background.

Laminated glass interlayers provide significant protection and energy dissipating capacity when windows are subjected to blast loads. Current protection criteria (DoD, GSA) prescribe protective glass assemblies and analytical approaches for validating protection using HazL (DoD) and WinGARD (GSA) glass response and hazard models. The laminated glass membrane models in these "standard" codes (HazL and WinGARD) used to predict glass laminate membrane response have been shown to yield both overly conservative as well as non-conservative results. Protection Engineering Consultants (PEC) has developed an improved and simple laminate membrane model that greatly increases the accuracy of hazard predictions. Static and dynamic laminated glass test data is available from tests in the last two to three years that will improve and validate the new variable modulus-based membrane model. The more accurate membrane model will provide cost savings through more efficient designs that meet protective glazing specifications and higher levels of confidence in protective system response.

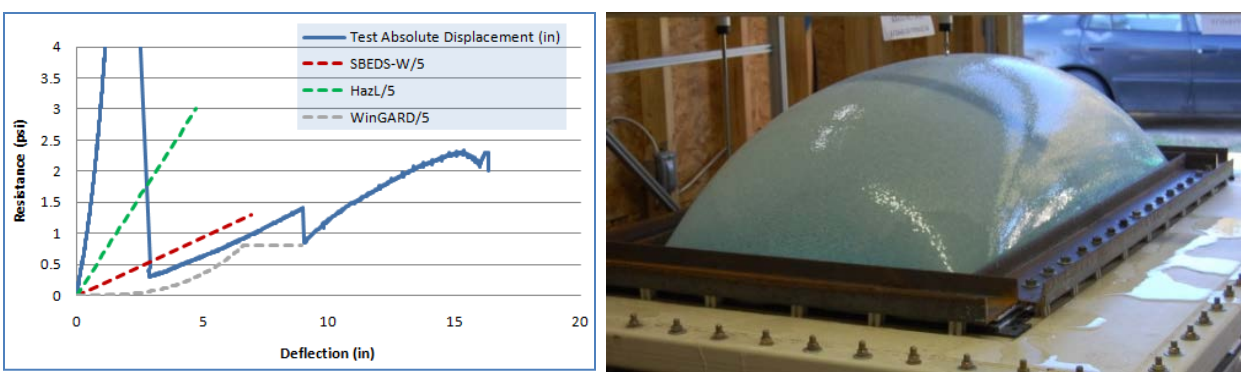

Figure 1-1. Comparison of Current Model Predictions for 64-in x 36-in Laminated Glass Window with 0.060-in PVB evaluated statically at PEC

Overview.

PEC has applied its direct experience with laminate glass static and dynamic tests, and has leveraged our relationships with US and UK agencies and laboratories to gather and use static and dynamic test data to improve and validate the glass laminate membrane model. PEC engineers have collected and evaluated recent test data on laminated glass specimens. PEC's variable modulus membrane model is being updated using this data; both for stiffness adjustments as a function of in-plane strain and for strain rate effects (dynamic increase factors) on membrane response. Following incorporation of all static and dynamic test data into the membrane model, a version of the improved model is being implemented into analysis tools such as Single Degree of Freedom Blast Effects Design Spreadsheet (SBEDS). The SBEDS platform is well-utilized by designers to perform SDOF dynamic analysis for other engineering materials (steel, reinforced concrete, masonry, etc…) and is well suited for incorporation of an additional material.

Data Collection.

Static and dynamic tests on laminated glass specimens have recently been conducted at the Corps of Engineers Engineering Research and Development Center at WES (ERDC) and at PEC in the US and at the Center for the Protection of the National Infrastructure (CPNI) in the UK. PEC has coordinated with engineers at these two agencies to collect all available recent data on laminate response. Static tests conducted at PEC on laminated glass specimens were included, as will test data already received from ERDC and CPNI. We will continue working with these agencies to collect data from tests currently in-process or recently completed. In addition, a literature review on PVB mechanical test data and dynamic response models will be conducted to further characterize the strain dependency of this material.

Evaluation and Incorporation of Data into a Membrane Model.

The new membrane model is based on a modified version of the theoretical equations for a two-way stretched rectangular membrane developed by Timoshenko (1959) and extended by Brokaw (presented in equation 1 below). The modifications account for the variability in Young's modulus as a function of in-plane strain as well as strain rate effects due rates of elongation during glazing response to blast loads. The resulting equations allow the membrane resistance to be calculated in terms of mid-span membrane deflection, thereby establishing a strain and rate dependant resistance-deflection relationship.

$$\sigma_x = \frac{E(\varepsilon_x + \upsilon \varepsilon_y)}{(1-\upsilon^2)}$$

$$\sigma_y = \frac{E(\varepsilon_y + \upsilon \varepsilon_x)}{(1-\upsilon^2)}$$

$$\varepsilon_x = \frac{-a\pi(2C_3 b^4 a^2 C_2 - C_5 C_4)}{4b^6 a^6 C_2 - C_5^{\ 2}} w_0^{\ 2}$$

$$\varepsilon_y = \frac{-b\pi(2C_4 b^2 a^4 C_2 - C_5 C_3)}{4b^6 a^6 C_2 - C_5^{\ 2}} w_0^{\ 2}$$

$$C_1 = (18b^2 a^2 + 81b^4 + 81a^4)\pi^4$$

$$C_2 = (10368 - 1152\upsilon)\pi^2$$

$$C_3 = (-3072a^3 + 768\upsilon ba^2 - 768ba^2 + 1536\upsilon b^2 a)\pi^2$$

$$C_4 = (-3072b^3 + 768\upsilon ab^2 - 768ab^2 + 1536\upsilon ba^2)\pi^2$$

$$C_5 = (16384\upsilon + 16384)b^3 a^3$$

$$p_0 = \frac{Etw_0^{\ 3}\pi^2}{73728}\left(\frac{4a^6 C_1 C_2^{\ 2} b^6 - a^6 C_4^{\ 2} C_2 b^4 - a^4 C_3^{\ 2} C_2 b^6 + C_4 C_5 C_3 b^2 a^2 - C_1 C_5^{\ 2}}{a^4 b^4 (1-\upsilon^2)(4b^6 a^6 C_2^{\ 2} - C_5^{\ 2})}\right)$$

Equation 1-1. Original Two-way Stretched Rectangular Membrane Equations per Timoshenko and Brokaw

Where:

a = 1/2 short dimension of membrane
b = 1/2 long dimension of membrane
v = Poisson's ratio
ε_x = maximum membrane strain in short span direction
ε_y = maximum membrane strain in long span direction
σ_x = maximum membrane stress in short span direction
σ_y = maximum membrane stress in long span direction
E = Young's modulus
t = membrane thickness
w_o = midspan membrane deflection
p_o = membrane resistance
$C_\#$ = constants

Variable Membrane Modulus—The results of various experimental test programs aimed at characterizing the response of glass laminate interlayers have suggested an in-plane strain dependency on the membrane modulus. It is hypothesized that this phenomenon is a direct consequence of the mechanical behavior of the elastomer interlayer material as well as the tension stiffening effect afforded by the portion of glass fragments that remain adhered to the interlayer at any given mid-span membrane deflection following the fracture of adjacent glass lites. In general, it is thought that as the in-plane membrane strain increases, the portion of glass fragments retaining their bond to the interlayer material decreases thus resulting in less tension stiffening. This decrease in tension stiffening coupled with material softening as the membrane stretches ultimately results in the loss of overall membrane stiffness.

Using the already obtained static test data, an equation for Young's modulus as a function of the in-plane membrane strain was determined by back-calculating a modulus, E_m, at each measured mid-span deflection, w_o, that would cause the resistance calculated with the theoretical membrane equations to match the measured resistance at each corresponding measured mid-span deflection. The back calculation procedure was implemented for each laminated window specimen. These calculated modulus values were then plotted against their associated theoretical long span membrane strain values, ε_y, and a nonlinear regression analysis was conducted based on a least squares error minimization methodology. Figure 1-2 illustrates the results of the nonlinear regression analysis from which an empirical equation for the variable membrane modulus was developed. It should be noted that polyvinyl butyral (PVB) was used as the laminate interlayer for all test data presented in Figure 1-2.

$$E_m(w_0) = 415.65\varepsilon_y(w_0)^{-1.005}$$

Equation 1-2. Empirical Variable Modulus Equation

Where: $E_m(w_0)$ = membrane modulus as a function of midspan membrane deflection
$\varepsilon_y(w_0)$ = long span membrane strain as a function of midspan membrane deflection
w_0 = midspan membrane deflection

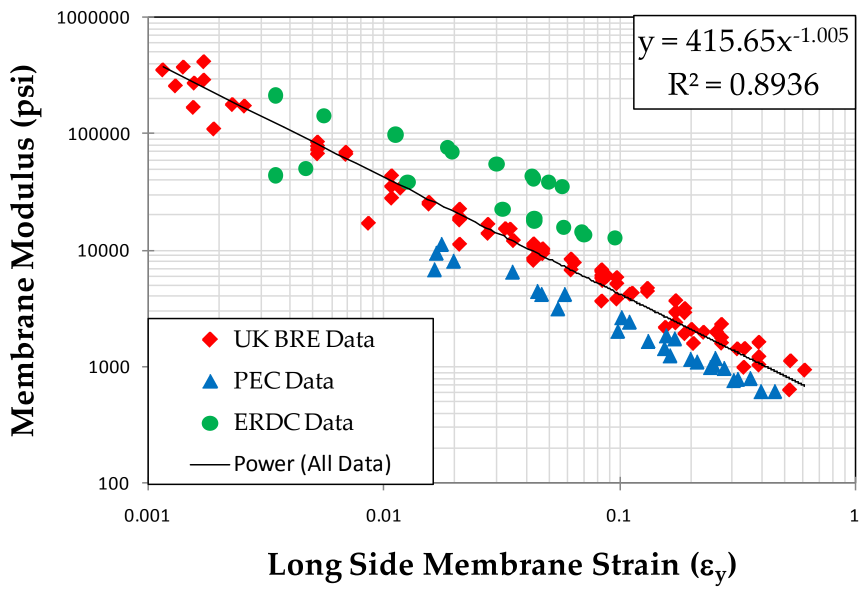

Figure 1-2. Results of Regression Analysis for Development of Refined Membrane Modulus Expression

It is clear from Figure 1-2 that a trend exists between membrane modulus and in-plane membrane strain. In particular, Figure 1-2 suggests a decrease in membrane stiffness as the in-plane membrane strain increases. This observed trend is consistent with PEC's initial hypothesis. In addition, this initial exercise demonstrates the need to incorporate the dependence of in-plane strain on membrane modulus into glass laminate interlayer response prediction methodologies. In order to develop a robust variable modulus equation for PEC's improved laminated glass membrane model, a more rigorous investigation of the mechanical behavior of PVB interlayer material and its interaction with adhered glass fragments must be conducted. This can be accomplished through the examination of additional experimental test results supplemented with a comprehensive literature review of relevant past research efforts aimed at investigating the mechanical behavior of PVB.

Strain Rate Effects—Laminate glass interlayers experience enhanced modulus/ultimate strength properties by virtue of the high strain rate induced during a dynamic loading scenario. In addition, the failure strain and associated deflection tend to decrease with increasing strain rate. The result is a stronger material that exhibits less ductility relative to its behavior during a quasi-static loading scenario. Therefore, the static resistance function for the interlayer must be modified for anti-terrorism and force protection (ATFP) applications to include a theoretically sound but empirically verified dynamic increase factor (DIF).

Using the already obtained dynamic test data, a procedure was developed for determining an appropriate DIF value for the PVB interlayer. A DIF was determined separately by trial and error for each specimen such that the maximum mid-span membrane deflection determined from an SDOF dynamic analysis would equal that from the measured test data. The DIF, which essentially controls the slope of the linear resistance of the laminate interlayer, was increased as necessary to accomplish this for each specimen. The results of the DIF calibration procedure suggest an average DIF of 3.60 and an average ratio of peak deflection to short side plan dimension of approximately 0.23. All of the blast test window specimens exhibited membrane response but did not fail, thus the aforementioned average DIF and peak deflection to short span ratio represent lower bound values.

The average strain rates depicted in Figure 1-3 were calculated by first estimating the in-plane membrane strain associated with the peak mid-span deflection by utilizing the theoretical membrane equations presented in Equation 1-1. The resulting strain was then divided by the time to peak mid-span deflection. An inherent assumption in this calculation procedure is a constant strain rate from initial air blast incidence (t = 0) to peak mid-span membrane deflection. Further investigation of the validity of this assumption is warranted through the examination of additional dynamic test data.

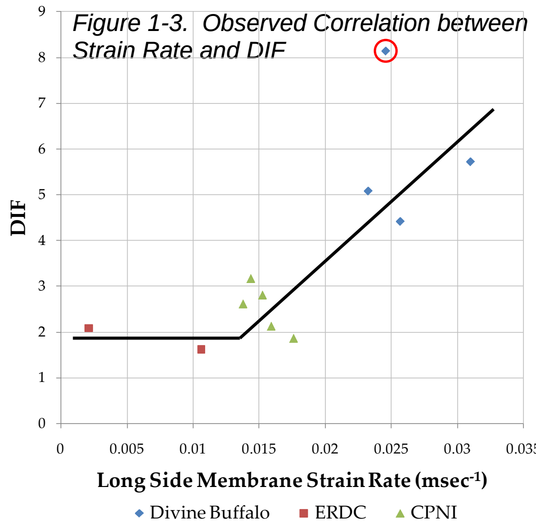

Figure 1-3. Observed Correlation between Strain Rate and DIF

In general, the data in Figure 1-3 suggest that an increase in strain rate may lead to an increase in the DIF. This behavior agrees well with past research efforts aimed at investigating strain rate effects of different materials (e.g., steel and concrete). However, there are not enough data points to make a definitive conclusion regarding this relationship. It should be noted that the Figure 1-3 data point circled in red represents a different window configuration than the rest of the data points. In particular, that particular test item was composed of three lites of annealed glass and two PVB interlayers, whereas the rest of the test specimens were composed of only two lites of annealed glass and one PVB interlayer. It is hypothesized that the relatively large calculated DIF for this test is a direct consequence of this altered window configuration. After the three lites of annealed glass fracture, the middle lite glass fragments become confined between two layers of PVB. This confinement effect promotes continued adhesion to the adjacent PVB interlayers longer into the membrane response than a typical two-lite window configuration. The result is a stiffer overall response due to the enhanced tension stiffening effect afforded by the confined glass fragments.

In addition to the lower bound estimates of DIF and peak mid-span deflection to short span ratio, upper bound estimates were made using the Hazard Level 3 specimens of the WINDAS database. First, the peak deflection to short span ratio was held constant at a value of 0.22 for all test cases, and a DIF was determined such that the peak mid-span deflection calculated from an SDOF dynamic analysis equaled the peak mid-span deflection estimated from this ratio. The average DIF was determined to be approximately 4.77; which proved to be about 32.5 percent larger than the average DIF determined from the experimental blast test data (DIF = 3.60). The Hazard Level 3

WINDAS specimens exhibited membrane response up to failure; therefore the calculated average DIF of 4.77 represents an upper bound value. A DIF of 5.0 was chosen to be implemented in the remainder of the study. Next, a peak deflection to short span ratio was assumed and the resulting resistance function was utilized in an SDOF dynamic analysis; all while holding the DIF constant at 5.0. This process was iterated upon for each specimen until the peak deflection determined from the dynamic analysis matched that from the assumed peak deflection to short span ratio. Upon conclusion of this study, the average peak deflection to short span ratio was determined to be approximately 0.21

Conclusion

The initial investigation of strain rate effects on the dynamic response of laminate interlayers resulted in upper and lower bound dynamic increase factor estimates, an appropriate peak deflection to short span ratio of approximately 0.22, and evidence of strain rate dependency on the dynamic increase factor. Based on these results, it is clear that the determination of an appropriate dynamic increase factor is crucial in ensuring high fidelity response predictions for laminated windows subject to an air blast. The development of a theoretically sound and more robust method for determining an appropriate DIF is deemed necessary, and this can be done through investigation and adaptation of existing rate dependent PVB material models and by acquisition of additional dynamic test data.